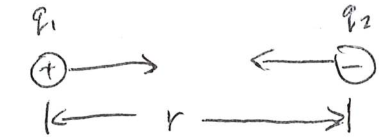
PHYSICS 121 | Midterm | Name: Sol'ns
Feb. 14, 2020 – 12:30 to 13:20 | Student Number:

Midterm (26 points)
Multiple Choice: Select the best answer for each of the following questions. Put a circle around your final answer.

(2pts) **1.** A positive and a negative charge are released from rest in vacuum. They move toward each other. As they do:

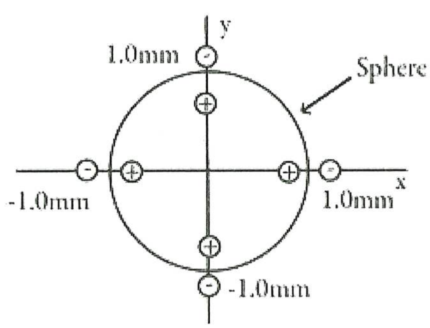
(a) A positive potential energy becomes more positive.
(b) A positive potential energy becomes less positive.
(c) A negative potential energy becomes more negative.
(d) A negative potential energy becomes less negative.
(e) The potential energy remains constant.



$U_e = \frac{K q_1 q_2}{r} < 0$ b/c $q_2 < 0$

when they move closer, r decreases.
∴ U_e becomes more neg.

(2pts) **2.** Four dipoles, each consisting of a $+10\,\mu C$ charge and a $-10\,\mu C$ charge, are located in the xy-plane with their centres 1.0 mm from the origin as shown in the figure below. The surface of a sphere passes through each dipole. What is the electric flux through the sphere?



$\Phi_e = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$

$Q_{in} = 4(+10\mu C)$ $\quad \varepsilon_0 = 8.85\times10^{-12}\ \frac{C^2}{Nm^2}$

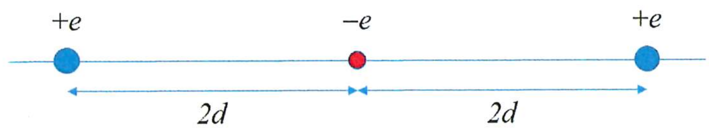
$\therefore \Phi_e = \frac{Q_{in}}{\varepsilon_0} = 4.5\times10^{6}\ \frac{Nm^2}{C}$

(a) $-1.1\times10^{6}\,\mathrm{Nm^2/C}$
(b) $-4.5\times10^{6}\,\mathrm{Nm^2/C}$
(c) $1.1\times10^{6}\,\mathrm{Nm^2/C}$
(d) $4.5\times10^{6}\,\mathrm{Nm^2/C}$
(e) The net electric flux through the sphere is zero.

4 pts

Free Response: Write out complete answers to the following questions. Include diagrams where appropriate. Show your work since it allows us to award partial credit.

(5pts) **3.** Initially, an electron (charge $-e$ and mass m_e) is at rest midway between two protons (charge $+e$ and mass m_p). For this problem, the two protons are fixed in place and the distance between them remains fixed at $4d$.



The electron is given a very slight nudge to the right. Find an expression for the speed of the electron when it is a distance d from the proton on the right. (**5 marks**)

$$U_i = -\frac{ke^2}{2d} - \frac{ke^2}{2d} + \frac{ke^2}{4d} = -\frac{ke^2}{d} + \frac{ke^2}{4d}$$

$$K_i = 0$$

$$U_f = -\frac{ke^2}{d} - \frac{ke^2}{3d} + \frac{ke^2}{4d} = -\frac{4ke^2}{3d} + \frac{ke^2}{4d}$$

$$K_f = ?$$

conservation of energy: $K_i + U_i = K_f + U_f$

$$\therefore -\frac{ke^2}{d} + \frac{ke^2}{4d} = \frac{1}{2}mv_f^2 - \frac{4ke^2}{3d} + \frac{ke^2}{4d}$$

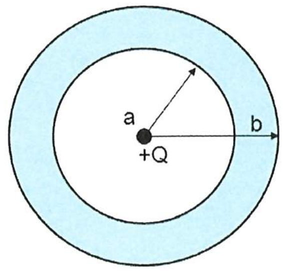
$$\therefore \frac{1}{2}mv_f^2 = +\frac{ke^2}{3d}$$

$$\therefore v_f^2 = \frac{2ke^2}{3md}$$

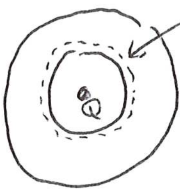
$$\therefore v_f = \sqrt{\frac{2ke^2}{3md}} = \sqrt{\frac{e^2}{6\pi\varepsilon_0 md}}$$

5 pts

(8pts) **4.** Consider a *conducting* spherical shell with inner radius a and outer radius b. A point charge $+Q$ is placed at the centre of the cavity.



(a) If the spherical shell is neutral, what are the charges at inner and outer surfaces of the shell? What is the magnitude of the electric field a distance $r > b$ from the centre of the shell? (**4 marks**)

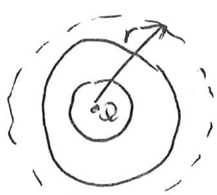


Gaussian surface in conductor where $\vec{E}=0$

$$\oint \vec{E}\cdot d\vec{A} = 0 = \frac{Q_{in}}{\varepsilon_0} \quad \therefore Q_{in} = Q_{inner} + Q = 0 \quad \therefore \boxed{Q_{inner} = -Q}$$

Because conductor is neutral, $Q_{inner} + Q_{outer} = 0$

$$\boxed{\therefore Q_{outer} = Q}$$



$$\oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E = \frac{Q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \qquad \boxed{\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$ like pt. charge.

(b) Now suppose the charge on the spherical conductor is $-Q$. If the shell still contains the $+Q$ point charge, what are the charges at inner and outer surfaces of the shell? What is the magnitude of the electric field a distance $r > b$ from the centre of the shell? (**4 marks**)

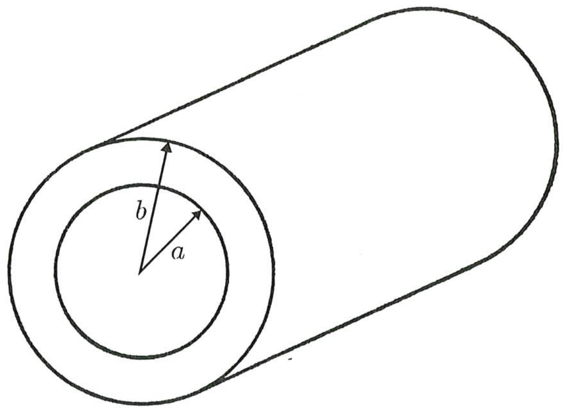
Like in (a) $\boxed{Q_{inner} = -Q.}$

charge on conductor $\underbrace{Q_{inner}}_{-Q} + Q_{outer} = -Q \quad \therefore \boxed{Q_{outer} = 0}$

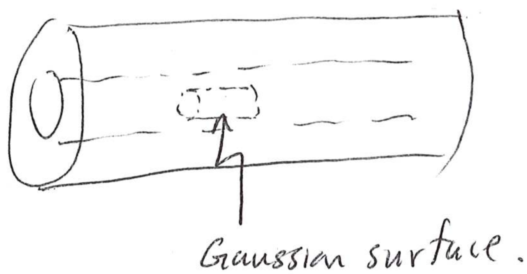
$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \quad \text{but } Q_{in} = 0 \quad \boxed{\therefore \vec{E} = 0}$$

8 pts

(9pts) **5.** Consider a very long uniformly charged hollow cylinder. The charge density is ρ and the cylinder's inner radius is a and its outer radius is b.



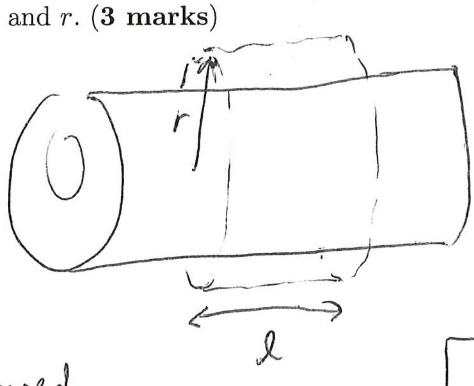
(a) Find the magnitude of the electric field a distance $r < a$ from the cylinder axis (i.e. at a point inside the hollow bore of the cylinder). Give your answer in terms of some or all of the variables ρ, a, b, and r. (**2 marks**)



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$$Q_{in} = 0 \quad \therefore \boxed{\vec{E} = 0}$$

(b) Find the magnitude of the electric field a distance $r > b$ from the cylinder axis (i.e. at a point outside the cylinder). Give your answer in terms of some or all of the variables ρ, a, b, and r. (**3 marks**)



$$Q_{in} = \rho \pi (b^2 - a^2) l$$

$$\oint \vec{E} \cdot d\vec{A} = E \, 2\pi r l$$

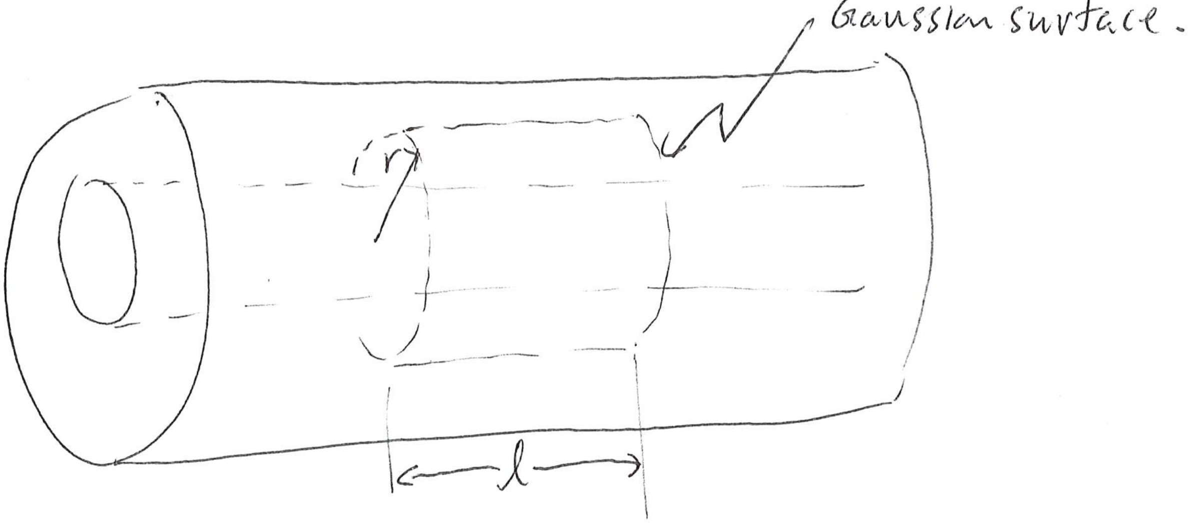
$$\therefore E \, 2\pi r l = \rho \pi (b^2 - a^2) l$$

$$\therefore \boxed{E = \frac{\rho (b^2 - a^2)}{2 \varepsilon_0 r}}$$

area of curved surface is $2\pi r l$

9 pts

(c) Find the magnitude of the electric field a distance $a < r < b$ from the cylinder axis (i.e. at a point within the material of the cylinder). Give your answer in terms of some or all of the variables ρ, a, b, and r. (**4 marks**)



Like in (b) $\oint \vec{E} \cdot d\vec{A} = E 2\pi r l$

$Q_{in} = \rho \pi (r^2 - a^2) l$

$\therefore \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \Rightarrow E 2\pi r l = \frac{\rho \pi (r^2 - a^2) l}{\varepsilon_0}$

$$\therefore E = \frac{\rho (r^2 - a^2)}{2 \varepsilon_0 r}$$

0 pts